# Comparison of an acousto-optic and a radiation force method of measuring ultrasonic power

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During the interim period while nationally acceptable methods of measurement of referential standards for ultrasonic power are being developed, comparisons are needed among measurment techniques currently used. This is particularly necessary for determining the acoustic power from ultrasonic medical devices. Such a comparison was made between an optical and a radiation force technique to measure the ultrasonic power output of a 1-MHz 1-in.-diameter PZT crystal from 70 mW to 2.3 W total acoustic power. The optical technique follows the theory of Raman and Nath, assaying the diffraction of monochromatic light by ultrasound. In addition, a correction factor is introduced into the calculation of the phase retardation parameter and eliminates the need to determine the pathlength of the light through the sound. The radiation force technique relates the change of position of an air-backed, self-centering, reflecting float to the ultrasonic power. A linear regression analysis between the square of the voltage applied to the crystal and the measured power of each method was used. For a 99% confidence level, there is no statistical difference between the measurements over the range of power investigated, although the variance of the optical measurements was significantly lower than that of the radiation force.

Subject Classification: 35.65, 35.80; 85.24.

## INTRODUCTION

During the interim period while nationally acceptable methods of measurement and referential standards for ultrasonic power are being developed, comparisons are needed among measurement techniques currently used. This is particularly necessary for determining the acoustic power from ultrasonic medical devices. These comparisons are an important step toward a referential measurement method. Current techniques include calorimetric[1-3] radiation force,[4,5] piezoelectric,[6,7] and optical methods.[8,9] A recently published comparison[10] determined the ultrasonic pressure amplitude using several methods. This paper presents a comparison between an optical and a radiation force technique for measuring ultrasonic power.

The optical technique employs the basic Raman–Nath theory in which it is assumed that the sound presents a phase grating to a normally incident beam of plane, monochromatic light. Based on a theoretical investigation[12] of the integrated optical effect of sound from a circular plane piston source, a correction was made for the axial distance from the face of the transducer to the point sampled by the light. This correction eliminates the need to know the pathlength of the light in the sound for the calculation of acoustic power.

The radiation force system is patterned after one developed by the Federal Institute of Physical Technology (West Germany)[5] for the evaluation and certification of medical ultrasonic instruments. It consists of a conically shaped air-backed reflector, the displacement of which is proportional to the incident acoustic power.

## I. THEORY

The general treatment of the ultrasonic diffraction of light is given by Raman and Nath[11] and others.[13,14] A beam of ultrasound produces a periodic density variation in the transmission medium. Collimated, monochromatic light is normally incident to the beam of ultrasound. The phase of the light is retarded proportionally to the local sound pressure. In the resulting Fraunhofer diffraction pattern, the $n$th-order normalized light intensities $I_n$ vary as

$$I_n = J_n^2(v), \tag{1}$$

where $J_n$ is the $n$th-order Bessel function and $v$ is a measure of the maximum optical phase retardation (known as the Raman–Nath parameter). This phase retardation is directly proportional to the local peak acoustic pressure. The determination of $v$ from the light intensities of each of the diffraction orders can be accomplished by using two or more diffraction orders (Eq. 1) and tables of values for Bessel functions. This technique assumes the acoustic waveform is a pure sinusoid. Bessel functions of argument $v$ are also defined by the recursion relation

$$v(J_{n-1} + J_{n+1}) = 2n\, J_n. \tag{2}$$

When the two equations above are combined, the parameter $v$ may be calculated from the normalized light intensities. It has been shown[15] that for certain conditions

  

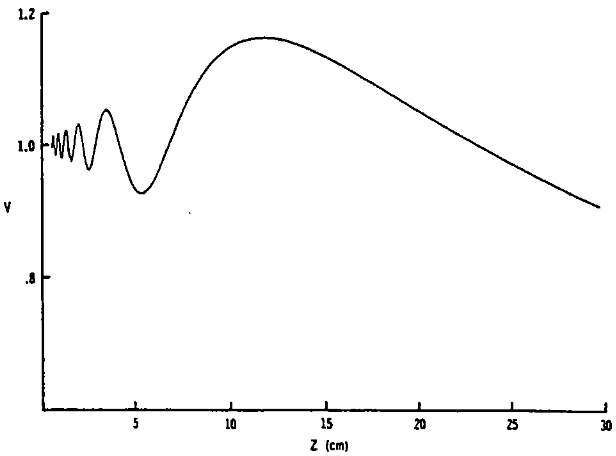


FIG. 1. Plot of the normalized Raman–Nath parameter ($v$) versus the axial distance ($z$) from a circular piston source of radius 1.27 cm having a frequency of 1 MHz.

of finite amplitude distorted ultrasonic waves Eq. 2 can be rewritten as

$$\sum_j j a_j v(\phi_{n-j} + \phi_{n+j}) = 2n\phi_n, \tag{3}$$

where $a_j$ is the ratio of the pressure in the $j$th harmonic to the fundamental and $\phi_n$ is the light amplitude in the $n$th diffraction order. This refinement of the basic recursion relation enables one to calculate the retardation parameter for harmonics as well as the fundamental. Information about the harmonic content aids the interpretation of the experimental data and is discussed in later sections. To calculate the phase retardation parameter $v$, let

$$A = (\phi_{n-j} + \phi_{n+j}), \tag{4}$$

$$B = (2n\phi_n), \tag{5}$$

and

$$V = (j a_j v), \tag{6}$$

so that Eq. 3 may be rewritten in the matrix notation as

$$AV = B. \tag{7}$$

The number of linear equations solvable for $a_j v$ exceeds the number of variables so $V$ may be obtained from

$$V = (A^T A)^{-1} A^T B, \tag{8}$$

which yields a least-squares fit to the data.

Relating the experimentally determined values of the phase retardation $v$ to the acoustic power requires knowledge of the influence the acoustic field radiated by a plane piston source has on the integrated optical effect. It has been shown[12] that for a circular piston of radius $a$ and centered on the $z$ axis, the optical phase retardation in the phase grating limit is given by

$$v(z) = v_0(ka/2)(\pi/2k\eta)^{1/2} J_0^2(k\xi) + J_1^2(k\xi)^{1/2}, \tag{9}$$

where

$$v_0 = 4\pi u_0 \rho_0 c a \kappa/\lambda_1, \tag{10}$$

$$\xi = [(z^2 + a^2)^{1/2} - z]/2, \tag{11}$$

$$\eta = [(z^2 + a^2)^{1/2} + z]/2, \tag{12}$$

and $J_0$ and $J_1$ are zero- and first-order Bessel functions, $k$ is the wavenumber of the sound. Equations 9 and 10 apply to a piston of velocity amplitude $u_0$ radiating into a semi-infinite nondissipative medium of density $\rho_0$ and having a sound velocity of $c$. $\lambda_1$ is the wavelength of light and $\kappa$ is the adiabatic piezo-optic coefficient. A normalized plot of $v(z)$ is found in Fig. 1. The term $v_0$ is the measure of the phase retardation at the source ($z = 0$).

With Eq. 9, one may experimentally determine $v$ at some finite distance from the face of a source and simply convert back to the value $v_0$ at the face of the crystal. The assumption that $ka$ is large and $p = u_0 c \rho_0$ permits $v_0$ to be written in the familiar form

$$v_0 = 2\pi L p \kappa/\lambda_1, \tag{13}$$

where $L$ is the source diameter ($2a$) and $p$ is the peak acoustic pressure. In previous expressions[11,13,14] of Eq. 13 $L$ represented the pathlength of the light through the sound and was usually measured from a schlieren photograph.[16] Finally,

$$P = (1/2\pi\rho_0 c)(a\lambda_1/2L\kappa)^2 v_0^2 \tag{14}$$

gives the total acoustic power output $P$ in terms of the measured parameter $v$.

It is important to note that at no time was any assumption made about the light–sound interaction length. The diameter of the transducer determines the magnitude of the integrated optical effect without affecting the amplitude or location of the relative maxima or minima. One does not need to measure the actual pathlength of the light through the sound but only the distance of the sampling light beam from the source along the acoustic axis and the dimensions of the radiating area.

The radiation force system schematically diagramed in Fig. 2 is used in our laboratory for the measurement of output from medical ultrasonic devices.[17] A thin conical reflector forms the cover of a hollow, air-filled cylinder, with a coaxial stem below. This float is weighted so that it is slightly heavier than water. When the stem is immersed in the beaker of carbon tetrachloride, neutral buoyancy is eventually achieved. When exposed to a vertically traveling beam of sound, the float is forced down farther in the carbon tetrachloride. The force $f$ (dynes) exerted on the float by sound of velocity $c$ (centimeters/sec) is related to the total acoustic power $P$ (watts) by

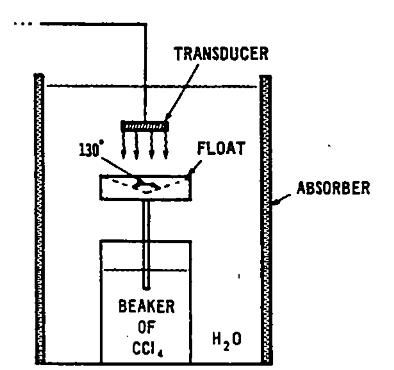


FIG. 2. Schematic diagram of the radiation force system.

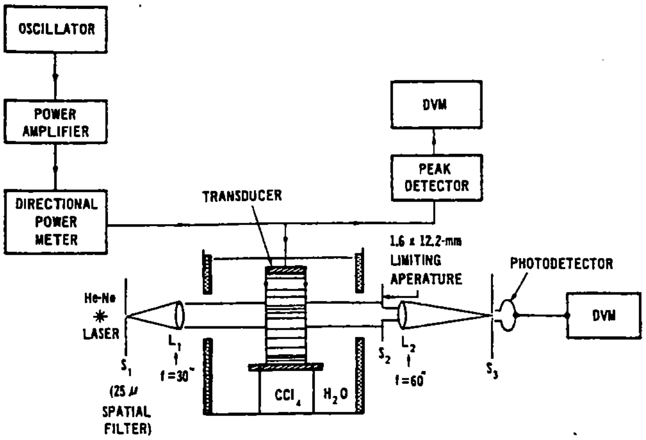


FIG. 3. Schematic diagram of the acousto-optic system and a block diagram of the generating electronics.

$$f=(2P/c)\cos^2\Theta(1.02\times10^2), \tag{15}$$

where $\Theta$ is the angle between the normal to the reflecting surface and the acoustic axis. The float was calibrated by using known weights and measuring the resulting displacement of the float.

## II. EXPERIMENT

A block diagram of the apparatus is shown in Fig. 3. The ultrasound was radiated into distilled, degassed water. Attenuation was disregarded because it is negligibly small compared to the correction factor of the Raman–Nath parameter. Windows were provided for the light from the laser. Except for a small opening for viewing, these windows were covered with sound-absorbing rubber during float (radiation force) measurements. During optical measurements, the beaker of $CCl_4$ was also covered with sound-absorbing rubber. This covering was angled so that any small impedance mismatch between the rubber and water did not result in reflections back toward the transducer. Progressive waves were verified by observing the normalized light intensity of the zeroth diffraction order as $v$ approached 2.4. For each optical measurement the zeroth order achieved its minimum value at less than $\frac{1}{2}\%$ of the normalized intensity of that order in the absence of sound.

Four trials of five power settings for each trial were performed for both the acousto-optic measurements and the radiation force measurements for a total of 40 different measurements. These power settings, ranging from 70 mW to 2.0 W acoustic power, were determined from values of the peak voltage applied to the crystal (1-MHz, 1-in.-diameter lead zirconate titanate). The voltage applied to the crystal was attenuated by a factor of 20, measured by a peak detector, and read on a digital voltmeter (DVM) allowing reproducible settings.

The following trial pattern was used for the experiment. Let $A$ represent optical measurements and $B$ represent float measurements. Voltage settings normally increased over a given trial; however, a prime (′) indicates voltage settings that were taken from high to low during a given trial. The trial order was $AA'BB'BB'AA'$. With this order, one could see short- or long-term drifts of the measured power with time.

To insure the perpendicularity between the optic and acoustic axes, the transducer alignment was adjusted by maximizing the normalized light intensity in the first diffraction order. After these initial adjustments, the transducer alignment was the same for all trials. All measurements were done in the one tank of water.

The light beam (1.6 mm×12.2 mm) sampled the sound field at 9.8 cm from the crystal. This location corresponded to the peak of the relative maximum farthest from the crystal where $v$ is calculated to be $1.16v_0$. Using a UDT-606 photodetector and digital voltmeter, normalized light intensities were measured in diffraction orders $-9$ to $+9$ during each optical trial for values of $v$ ranging from 1.0 to 4.5. The float was calibrated with weights after each float trial. After the data were taken, a Hewlett–Packard 9820 programmable calculator was used to find the Raman–Nath parameter $v$, the correction factor for $v$, the power levels for the optical measurements, and to perform the statistical analysis.

## III. RESULTS AND DISCUSSION

The data obtained are plotted in Fig. 4. Table I contains the statistical analysis of these data for the optical and the float measurements. The phase retardation parameter for the second harmonic of the acoustic wavefront was calculated for each optical measurement and a ratio formed with the parameter calculated for the fundamental. This ratio is equivalent to a ratio of the pressure amplitudes and never exceeded 1.0%. Under this condition, it can be said that the acoustic power measured is directly proportional to the square of the voltage applied to the crystal. Harmonic content of the acoustic wavefront is important for the radiation force measurements in that the greater absorption of the higher harmonics depletes the total power measured by the float. This depletion increases with intensity (i.e., particle displacement amplitude) as well as distance from the source of sound.

A linear regression analysis[18] based on constant error variance was used to analyze the measured power as a function of the square of the voltage. The peak voltmeter provided rigorous control of the applied voltage so variations are due to random or systematic differences within or between measurement techniques for the acoustic power. Thus, the "population variance" listed in the table refers to the estimated error variance of repeated power measurements at any given input voltage

Referring to the analysis table, the difference between the slopes for each technique is 0.072 $W/V^2$ and the ratio of the variances of the slopes is 32.1. Because of this significantly large $F$ ratio a modified $t$ test must be performed to compare the slopes obtained for the two measurement methods. The $t$ value for differences between the slopes is 2.3. A magnitude of $t$ as large or larger would occur by chance in 3% of the cases for the modified number of degrees of freedom (19) in the present experiment. This is to say that, for a 95% confidence level, the slopes are statistically different, but for a 99% confidence level, there is no statistical difference,

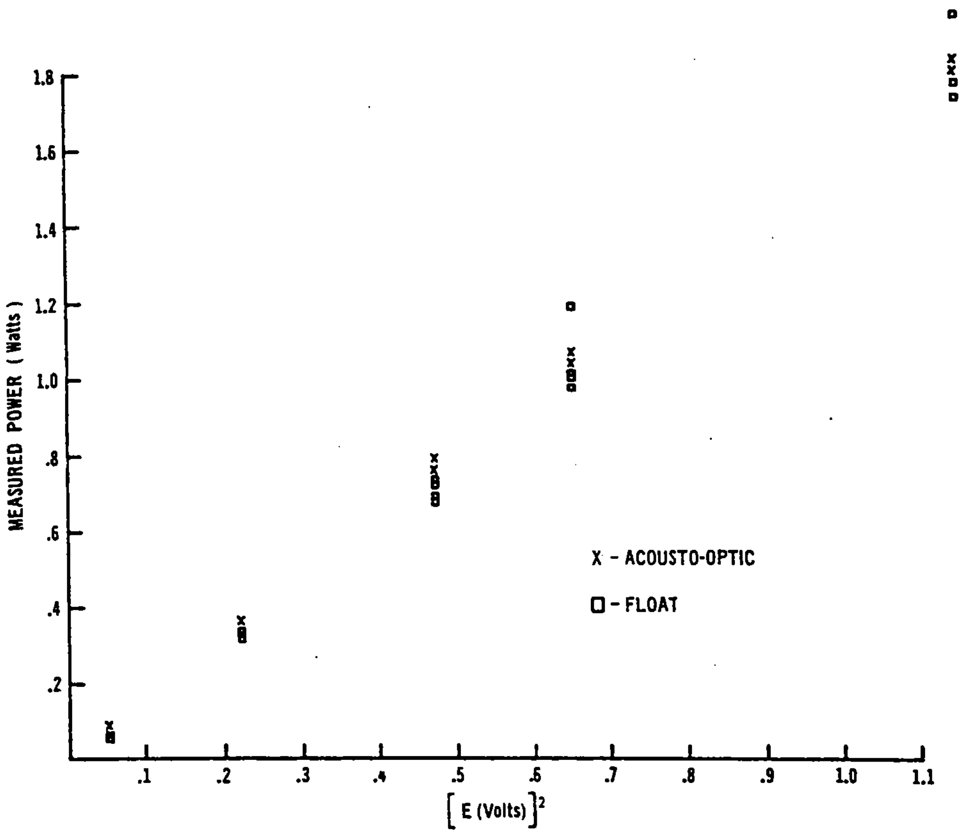


FIG. 4. Plot of the acoustic power measured by each method versus the square of the voltage applied to the crystal. Measured voltage is attenuated by 20.

or the significance of the difference between the slopes of each measurement method depends upon the confidence level desired.

Not considered yet have been the calibration constants for each method. For the float system, the multiplicative calibration constant for conversion from centimeters of displacement to watts of acoustical power is determined experimentally. It is found by dropping known weights on the float and measuring the resulting displacement. This result, expressed in centimeters/g, is then converted into centimeters/W knowing the cone angle of the float. The calibration constant was so determined after each of the four float trials, and each was used with the corresponding trial to convert float displacement to acoustic power. The mean of these calibration constants was 2.39 cm/W; the standard deviation was 0.104 or 4.4% of the mean. This represents an error of reproducibility of the calibration constant over and above the errors of reproducibility in the replications of the float data.

As for the optics, Eq. 13 converts the experimentally determined $v_0$ into $p$, the peak acoustic pressure. $L$, the physical diameter of the radiator, can be accurately measured in order to preclude any question about the size of the acoustic field through which the light passes. The adiabatic piezo-optic coefficient was found to be $14.65\times10^{-12}$ $cm^2$/dyne by Raman and Venkataraman[19] and is said to be "probably reliable to about 0.1%." More recent absolute determinations of this coefficient have not been done although relative values have been determined.[20]

Considering the magnitude of the standard deviation of the float calibration constant, it cannot be conclusively stated that the difference between the slopes of these methods shows a systematic difference over the range of powers investigated. More study needs to be done in a redetermination of the piezo-optic coefficient, a more accurate calibration of the float system, and more replications over the desired range of power.

Accurate and reproducible measurement of acoustic power is necessary for precise dosimetry in medical and research programs. While referential standards for

TABLE I. Statistical analysis of the data obtained from the optical and radiation force techniques.

| | Slope (watts/$V^2$) | Intercept (watts) | Population variance | Slope variance | Intercept variance |
|---|---|---|---|---|---|
| Acousto-optic measurements | 1.589 | $9.61\times10^{-3}$ | $8.36\times10^{-5}$ | $2.92\times10^{-5}$ | $1.92\times10^{-5}$ |
| Float measurements | 1.517 | $-3.46\times10^{-3}$ | $2.25\times10^{-3}$ | $9.34\times10^{-4}$ | $5.64\times10^{-4}$ |

the measurement of ultrasonic power are being developed, the question of absolute accuracy remains and requires comparisons between techniques to establish the degree of agreement. The comparison presented in this paper indicates good agreement between two fundamental techniques over the range investigated. In addition, the variance of the float measurements, being much greater than the optics measurements, indicates the optical technique is significantly more reproducible and lends itself to a calibrating measurement system. However, the float (radiation force) system lends itself more readily to measuring sources of unknown geometry, such as medical ultrasonic devices.


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