# KLM TRANSDUCER MODEL IMPLEMENTATION USING TRANSFER MATRICES

A. R. SELFRIDGE[1] and S. GEHLBACH[2]

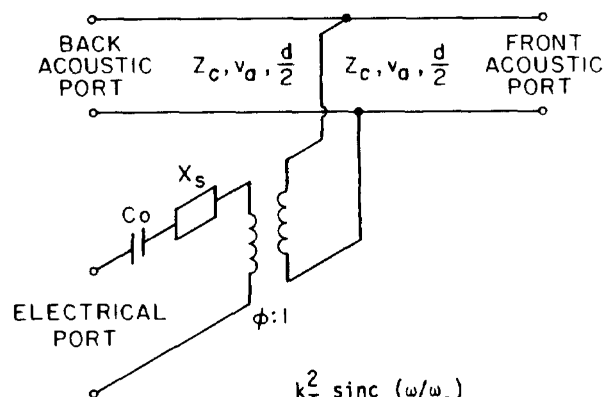
[1]Ultrasonic Devices Inc., Palo Alto, CA and [2]Kesa Corp., Santa Clara CA.

The Krimholtz, Leedom, and Matthaei transmission line model for acoustic transducers has been implemented on an IBM PC-XT computer using transfer, or "ABCD" matrices. Use of matrix formalism makes the programs easy to understand and allows quantities such as voltage, stress, strain and current to be readily calculated anywhere in the model. Previously difficult to model cases, such as open circuit receivers, are much simpler to treat with the new formalism. Furthermore, an accurate account of total linear phase is made, and acoustic and electrical loss is properly treated to first order.

## 1. Introduction

The KLM and Mason models have proven to be very useful to designers of devices utilizing the piezoelectric properties of materials. If one is given the area, thickness, various electroacoustic properties and boundary conditions on a piezoelectric material, it is possible to predict the electrical impedance into a pair of electrodes across the piezoelectric material. Furthermore, if one also knows about the electronic circuit which is connected to the electrodes, then it is also possible to predict the electro-acoustic conversion efficiencies and bandwidths for the overall system with these models.

Computer programs have been written by many separate researchers for the purpose of implementing the KLM model as an aid to designing transducers. At least two such researchers have published their programs [1,2]. The programs by Fraser and DeSilets have found many widely diversified applications, however, they utilized a power flow analysis formalism. This formalism, while it results is an elegantly simplified analysis in certain circumstances, does not lend itself well to many other applications. The programs became quite cumbersome when modified by Baer to treat acoustic loss, and required a number of interesting assumptions to be made to calculate one way transfer functions through devices. Also, the Fraser and DeSilets implementation did not keep track of total linear phase, which can be useful if one is designing a Fresnel lens or other device where time delay is important.




$$X_s = \frac{k_T^2 \,\mathrm{sinc}\,(\omega/\omega_0)}{\omega C_0}$$

$$\phi = k_T \sqrt{\frac{\pi}{\omega_0 C_0 Z_C}}\ \mathrm{sinc}\left(\frac{\omega}{2\omega_0}\right)$$

Figure 1. KLM model.

To overcome these shortcomings, the programs have been completely rewritten using a transfer matrix formalism which will be discussed shortly. In view of the greatly enhanced power of modern computers, the programs have been written for the analyst, not in some cryptic way to reduce execution times. We have found that the readability of the model has been greatly aided through the use of transfer matrices. This paper will discuss our implementation, and show examples of how it was used to model an acoustic hydrophone designed for power output measurements.

## 2. The T Matrix Implementation

The T-matrix or transmission matrix method of circuit analysis, is extremely convenient for the analysis of transducer networks. The resulting T-matrix for a cascaded network is merely the product of the individual T Matrices. This method, also known as the method of the ABCD parameters, is extensively described in Kuo (pg. 262-266), and briefly reviewed as follows.

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The four elements of the 2x2 T matrix are defined as shown in Fig. 2. For example, a simple series complex impedance has parameters A,D=1; D=Z; and C=0. A parallel (shunt) impedance is characterized by A,B=1; C=1/Z, B=0. A transmission line segment has A,D=cosh($\gamma$x), B=Zosinh($\gamma$x), C=(1/Zo)sinh($\gamma$x), where $\gamma$ is the propagation constant, and given in [4] (pg. 22-4). A ladder network can be analyzed by multiplying the T matrices of each element. The T-matrix method easily accounts for lossy components, which is important for accurate transducer designs.

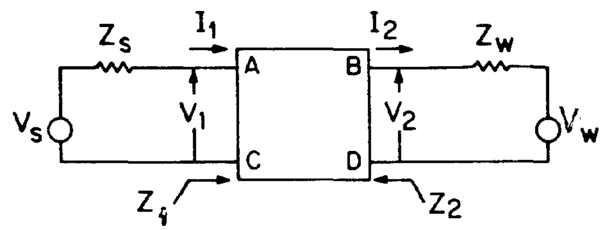
The T matrix carries with it a complete representation of the two port network. Important network parameters can be found as combinations of the ABCD elements, yielding any of the Z,Y,S, or H parameters (See Kuo, pg. 266). For example, by inspection of the definition in Fig. 2, the voltage transfer function (output open circuited) is 1/A. The input impedance, output open, is A/C, etc.

To apply the T matrix concept to the KLM model, shown in Fig. 1, observe first that the model can be analyzed as a ladder network between the front acoustic port and the electrical port, with four different types of segments in the ladder. Starting from the front acoustic load, there are transmission line segment(s), then a shunt element segment for the back acoustic load impedance, next a segment for the "ideal" transformer (with a complex turns ratio), and finally a segment with series capacitance. The program "Xducer", available from the authors, calculates the T matrix for each one of these segments, then multiplies them together a segment at a time to obtain the T matrix for the overall two port KLM model. The program can handle up to nine matching layers between the acoustic loads and the piezoelectric layer, each of which is modeled as a transmission line segment between the ports shown in Fig. 1 and the acoustic load. We have chosen to calculate the T matrix at 128 frequencies from f0/64 to 2*f0 where f0 is defined as the frequency at which the piezoelectric layer is half a wavelength, assuming open circuit conditions (D = 0) on the resonator. All units are kept in MKS within the programs, though not necessarily between programs, where units are chosen to be "user friendly".

One very important subtlety, which was discovered late in the development of the programs, is the fact that the KLM model associates the electrical values at the electrical port of the device with FORCE and velocity at the acoustic port. This means that instead of using Rayl units for impedance, it is necessary to use Rayls times area units. Normally acoustic impedance is a ratio of stress and velocity, here we must think in terms of "mechanical" impedances, or ratios of force to velocity. This only became apparent when the models were first used to predict oneway transfer functions through acoustic devices.

Once the T matrix is known for the transducer, the electrical impedance into the electrical port is easily calculated given the acoustic load on the front port. When the termination on the electrical port is defined, a program can be written to calculate the overall electrical to acoustic transfer function for the transducer system. With this, quantities such as bandwidth, insertion loss and impulse response can be obtained which greatly aid in design. A large series of programs have been written which use the T matrix of the transducer, calculated by Xducer, to plot various quantities of interest.

A fairly detailed description of how the T matrix approach is used to model hydrophones is given in Part 3. A very similar analysis was used to evaluate various piezoelectric materials for their inherent signal to noise ratio properties. Another project used the models to calculate the acoustic impedance seen looking into the front acoustic port, used to evaluate its reflection coefficient as a function of frequency. Other programs have been written which account for tone burst excitation of a transducer. Finally, a whole series of programs have been written which are used to model phased array elements. In this case, the acoustic load impedance put on the front acoustic port is complex, owing to the fact that the acoustic aperture is typically less than a wavelength (in the load) wide. Eventually we plan to rewrite the transducer optimization programs to use the T matrix formalism.




$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \qquad D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$

Figure 2. T matrix model for hydrophone

## 3. Application of Models to Hydrophone Design

One area of great interest lately is the design and performance of small, broadband hydrophones for acoustic output measurements and other applications. This is a good example where T matrix analysis is an excellent approach to the modeling problem. A typical hydrophone system is designed to operate at frequencies well below resonance, and into receivers with very high impedance inputs to obtain the flat frequency characteristics. The modeling problem calls for a oneway transfer function relating acoustic intensity to output voltage. None of the above considerations lends itself well to a power flow analysis, but does not complicate the T matrix approach at all. We picture the hydrophone as a network matrix as shown below in Fig. 2. where Zw is the acoustic impedance of water times the area of the hydrophone, $Zs$ is the input impedance of the receiver preamplifier, and $V_w$ is the acoustic force in water. The ratio between acoustic force in the load to voltage at the receiver input is obtained as follows.

Note the slightly different sign convention used here as compared to that used by Kuo having to do with I2. Because the transducer network can be shown to be reciprocal, calculating the effect of the output on the input is identical to calculating the effect of the input on the output. Therefore we can set $V_w=0$, and calculate the effect of $V_s$ across $Z_w$. Since I2 = V2/Zw and V2 = I2*Zw we obtain,

$$V_1 = AV_2 + BI_2 = V_2\left(A + \frac{B}{Zw}\right) \quad (1)$$

$$I_1 = CV_2 + DI_2 = I_2(D + CZ_w) = \frac{V_s}{Z_s + Z_1} \quad (2)$$

where $Z_1$ is defined as $V_1/I_1$. Solving Eq.2

for $\frac{V_s}{I_2}$ we obtain

$$\frac{V_s}{I_2} = (Zs+Z_1)(D+CZw) = Zs(D+CZw) + (AZw+B) \quad (3)$$

By reciprocity we have $\frac{V_2}{I_2} = \frac{-V_w}{I_1}$ and since

$$\frac{V_w}{V_1} = \frac{V_1}{-IZs} = \frac{1}{Zs}\left(\frac{-V_2}{I_1}\right) \quad (4)$$

we have the desired result $\frac{V_1}{V_w} = \frac{Z_s}{\left(\frac{V_s}{I_2}\right)}$ . (5)

Eq. 5 is used to plot the voltage output by a hydrophone as a function of frequency assuming various receiver input impedances. The result is shown in Fig 3 for a hydrophone constructed with PZT5A, backed by a steel rod, having a diameter of 0.016 inches and a thickness of 0.0043 inches. The acoustic stress exciting the hydrophone was assumed to have an intensity of 1 mW/cm**2. Note that the model assumes the resonator of a transducer is single moded, i.e. it does not account for any lateral mode coupling. This is of course important for PZT resonators with this geometry, but is ignored at this time to examine the effect of receiver impedance on hydrophone, frequency response.

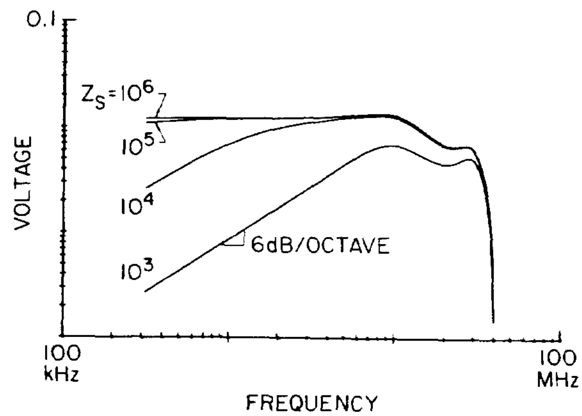



Figure 3. Voltage response of hydrophone.

Similar results can be obtained to examine the effect of loading the hydrophone with a length of cable between it and the receiver. In this case the T matrix for the transducer would first be multiplied by the T matrix for the cable (loss included if desired), and then an analysis identical to the above would be done. While it is not necessary to show all these results here, the curious reader may like to know that 8" of RG 174 cable acted like a 20 dB attenuator on the hydrophone and did not significantly effect the shape of the frequency response.

## 4. References


[1] Fraser, J. D., Ph.D. thesis, The Design of Efficient, Broadband Ultrasonsic Transducers, Ginzton Lab Report No. 2973, Stanford University, May, 1979

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[3] Kuo, F. F., Network Analysis and Synthesis, Second Edition, John Wiley & Sons, 1962

[4] References Data for Radio Engineer, Fifth Edition, Howard W. Sams & Co., Inc., 1968